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STRESS INTENSITY ESTIMATES FOR A PERIPHERALLY CRACKED SPHERICAL VOID AND A HEMISPHERICAL SURFACE PIT

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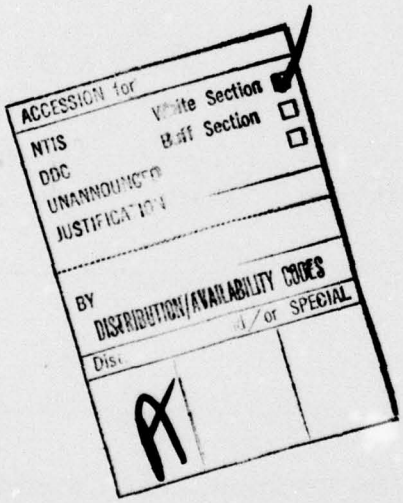
ABSTRACT

Stress intensity factors are estimated for three-dimensional defects which occur in ceramic bodies. The two idealized cases considered are: (A) a spherical void with a circumferential crack at its equator stressed by uniaxial tension at infinity and (B) a hemispherical pit at a free surface of a semi-infinite body also stressed by uniaxial tension with a circumferential crack at the semiequator of the pit. These stress intensity factors, which are given as a function of the crack length L to radius R of the spherical void or hemispherical pit, are considered as estimates because of the approximate nature of the analysis.



INTRODUCTION

Researchers in recent years, such as Evans and Tappin,[1] Molner and Rice,[2] Baratta et al.,[3] and Bansal and Duckworth,[4] to name a few, have attempted to utilize linear elastic fracture mechanics to determine fracture energy or mode I fracture toughness (K_{IC}) of ceramic materials from individual tension or bend specimens by strength and flaw characterization. This attractive approach, if accurately accomplished, is worthwhile pursuing because it will provide in situ K_{IC} data from a simple specimen configuration. However, it is difficult to accurately assess stress intensity factors associated with even regularly-shaped single inherent flaws in ceramic materials because of the complexity of three-dimensional mathematical analyses. Only a limited number of such solutions are available. Although there are other reasons why stress intensity factors associated with inherent flaws in materials are ill defined, such as irregular shape, defects distribution, etc., only those single flaws of regular shape which do occur[4] in ceramic materials, see Kirchner et al.[5] also, are considered here. Such results, although approximate, and heretofore unavailable, will aid in fracture mechanics applications and provide guidance until more rigorous analyses become available.

Therefore, the object of this paper is to estimate stress intensity factors for (a) a spherical void with a circumferential crack at its equator in an infinite body under uniaxial tensile stress, as shown in Figure 1, and also include

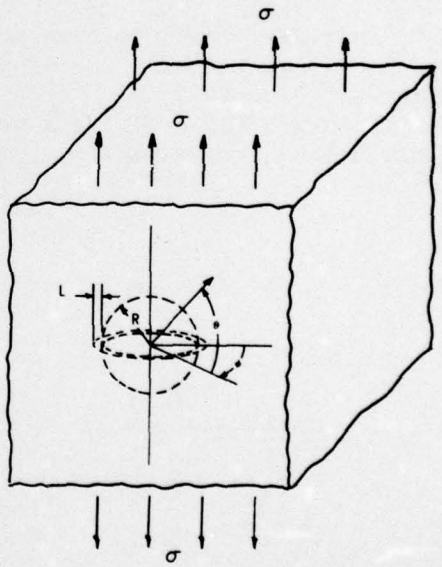


Figure 1. A spherical void with a circumferential crack at its equator.

1. EVANS, A. G., and TAPPIN, G. *The Effects of Microstructure on the Stress to Propagate Inherent Flaws.* Proc. Br. Ceram. Soc., no. 20, 1972, p. 275-297.
2. MOLNER, B. K., and RICE, R. W. *Strength Anisotropy in Lead Zircate Titanate Transducer Rings.* Am. Ceram. Soc. Bul., v. 52, no. 6, 1973, p. 505-509.
3. BARATTA, F. I., DRISCOLL, G. W., and KATZ, R. N. *The Use of Fracture Mechanics and Fractography to Define Surface Finish Requirements for Si_3N_4 in Ceramics for High Performance Applications.* J. J. Burke, A. E. Gorum, and R. N. Katz, eds., Brook Hill, Chestnut Hill, Massachusetts, 1974.
4. BANSAL, K., and DUCKWORTH, W. H. *Fracture Stress as Related to Flaw and Fracture Mirror Sizes.* J. Am. Ceram. Soc., v. 6, nos. 7-8, 1977, p. 304-310.
5. KIRCHNER, H. P., GROVER, R. M., and SOTTER, W. A. *Characteristics of Flaws at Fracture Origins and Fracture Stress-Flaw Size Relations in Various Ceramics.* Mtls. Sci. and Eng., no. 22, 1976, p. 147-156.

(b) a circumferentially-cracked hemispherical pit at a free surface of a semi-infinite body also stressed by uniaxial tension, as shown in Figure 2. Stress intensity estimates for these two cases will be developed.

PROCEDURE

The procedure is one of applying the appropriate stress, compatible with the defect discontinuity, to the crack tip for the known limiting cases of stress intensity and utilizing an interpolation scheme between limits to determine intermediate values. According to Barenblatt,[6] the stress intensity can be obtained by loading the crack faces with the negative of the stresses that would normally exist on the plane of the crack, if the crack was absent. However, in the analyses to follow, rather than apply the stress distribution to the crack face, only the stress caused by the defect discontinuity will be applied to the crack tip as suggested by Cartwright.*

In order to demonstrate the details of the procedure and evaluate the validity of the approach, it will be applied to a problem already solved by more accurate means. Consider a crack in a region of high stress concentration, such as a single radial crack originating from a circular hole in an infinite plate subjected to uniaxial uniform tension at infinity, as shown in Figure 3a. This problem has been successfully solved by Bowie[7] and will be used to evaluate the following approach.

We initially consider the crack length L to be very small compared to the hole radius R. Since the hole radius is very large compared to the crack length we can consider it to be at an edge of a semi-infinite plate, as shown by the equivalent system in Figure 3b, but with a stress distribution $\sigma_\theta(r)$ that arises because of the hole. The normalized stress intensity ratio $K_I/\sigma\sqrt{\pi L}$ for an edge crack in a

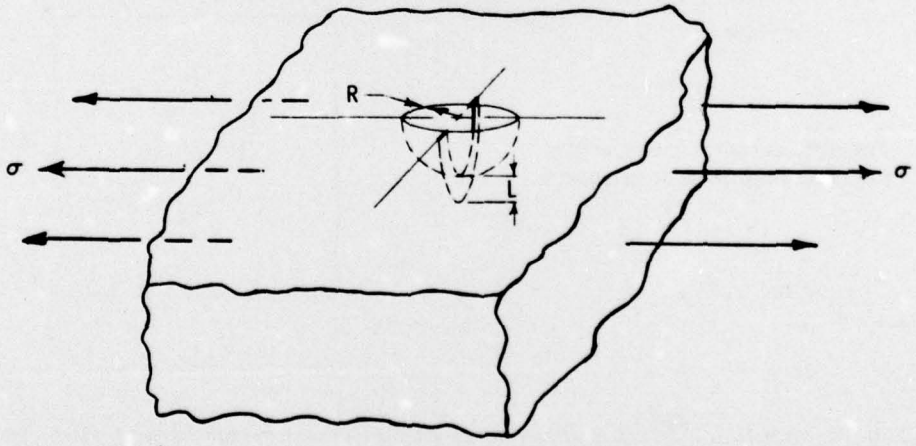


Figure 2. A hemispherical surface pit with a circumferential crack at its semiequator.

*CARTWRIGHT, D. J., University of Southampton, England, Internal Publication.

6. BARENBLATT, G. I. *The Mathematical Theory of Equilibrium Cracks in Brittle Fracture.* Advances in Applied Mechanics, v. 7, Academic Press, 1962.

7. BOWIE, O. L. *Analysis of an Infinite Plate Containing Radial Cracks Originating at the Boundary of an Internal Circular Hole.* J. Math. Phys., v. 35, no. 11, 1956, p. 60-71.

semi-infinite plate stressed in tension at infinity given by Paris and Sih[8] is $K_I/\sigma\sqrt{\pi L} = 1.12$, where σ is the remotely applied uniform tensile stress. At the opposite extreme when the crack length is very large compared to the hole radius, i.e., $L/R \to \infty$, the normalized stress intensity ratio $K_I/\sigma\sqrt{\pi L} = 0.707$, from Reference 7. It appears that this ratio is dependent upon some function of L/R which has limits of 1.12 when L/R = 0 and 0.707 when $L/R \to \infty$. Thus, intermediate values of the normalized stress intensity ratio can be approximated by choosing a function which is well behaved between the above stated limits. The function arbitrarily chosen for this problem as well as those to follow is of the form:

$$f(L/R) = c - k(\tan^{-1} L/R)^m. \tag{1}$$

Thus, for a circular hole with one radial crack the normalized stress intensity ratio is:

$$K_I/\sigma_\theta(r)\sqrt{\pi L} = f(L/R) = c - k(\tan^{-1} L/R)^m, \tag{2}$$

where $\sigma_\theta(r)$ is the stress distribution caused by the circular hole, provided by Timoshenko and Goodier,[9] which is

$$\sigma_\theta(r) = \sigma[1 + (1/2)(R/r)^2 + (3/2)(R/r)^4], \; r \geq R, \tag{3}$$

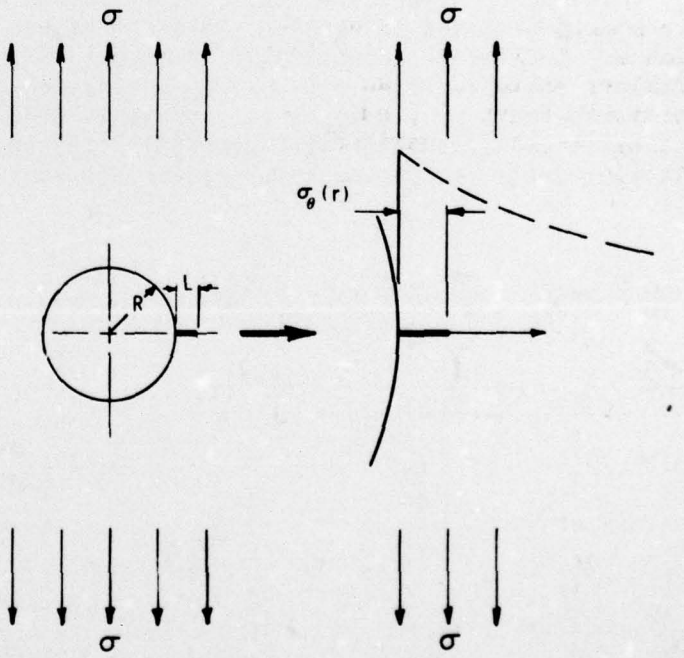


Figure 3. Circular hole with a single radial crack.

a. Cracked Circular Hole b. Equivalent Cracked Circular Hole

8. PARIS, P. C., and SIH, G. C. *Stress Analysis of Cracks.* ASTM-STP 381, 1970, p. 30-83.
9. TIMOSHENKO, S., and GOODIER, J. N. *Theory of Elasticity.* McGraw-Hill Book Company, Inc., 2nd Ed., 1951.

where r is the radial distance from the center of the hole. Substitution of Equation 3 into Equation 2 and transforming the coordinate axis r by R + L, gives:

$$K_I/\sigma\sqrt{\pi L} = [c - k(\tan^{-1} L/R)^m]\left[1 + (1/2)\left(\frac{1}{L/R + 1}\right)^2 + (3/2)\left(\frac{1}{L/R + 1}\right)^4\right]. \qquad (4)$$

Equation 4 represents the estimated normalized stress intensity ratio $K_I/\sigma\sqrt{\pi L}$ for a radial crack of length L emanating from a circular hole of radius R. The particular constants which appear in Equation 4 are readily determined by knowing c = 1.12 when L/R = 0; k and m are found by allowing f(L/R) = 0.707 when L/R = ∞, and f(L/R) = 0.94 when L/R = 3.0 (taken from Reference 7). The latter value was chosen so that the error span on the function would be minimized throughout the total range of L/R. Thus, c = 1.12; k was found to be 0.119 and m to be 2.748.

Equation 4 was then utilized to obtain $K/\sigma\sqrt{\pi L}$ as a function of L/R from 0 to infinity. It is seen that these results shown in Table 1 are within ±6% when compared to the more accurate data of Reference 8. Although the results of this procedure are approximate, it is believed that the errors involved when applied to the problems represented by Figures 1 and 2 will be of similar magnitude as those given in Table 1. Thus, a similar approach is used to solve these problems of interest.

Peripherally-Cracked Spherical Void

Goodier[10] provides the stress distribution for a spherical defect in an infinite medium stressed in tension. Because of the defect three stresses are developed, which are indicated schematically in Figure 4; they are a radial stress σ_r acting normal to the interface; a stress σ_θ acting in a tangential direction to a meridian (the north-south pole axis of the sphere is considered aligned with the axis of the applied stress); and another tangential stress σ_ϕ normal to both σ_r and σ_θ. In the following analysis, the maximum tensile stress is assumed to cause crack initiation.

Table 1. COMPARISON OF APPROXIMATE TO EXACT STRESS INTENSITY RESULTS FOR A CIRCULAR HOLE WITH A RADIAL CRACK

	$K_I/\sigma\sqrt{\pi L}$		
L/R	From Eq. 3	Reference 7	% Difference
0	3.36	3.39	-1
0.1	2.73	2.73	0
0.2	2.32	2.30	+1
0.3	2.04	2.04	0
0.4	1.83	1.86	-2
0.5	1.69	1.73	-2
0.6	1.56	1.64	-5
0.8	1.40	1.47	-5
1.0	1.29	1.37	-6
1.5	1.13	1.18	-4
2.0	1.03	1.06	-3
3.0	0.94	0.94	0
5.0	0.86	0.81	+6
10.0	0.78	0.75	+4
∞	0.708	0.707	0

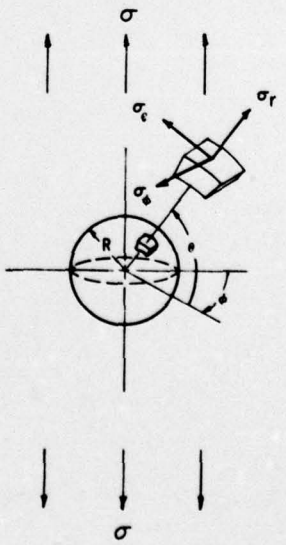


Figure 4. Coordinate system to describe the stress state around a spherical void.

10. GOODIER, J. N. *Concentration of Stress Around Spherical and Cylindrical Inclusions and Flaws.* Trans. Am. Soc. M. E., v. 55, 1933, p. 39-44.

It is indicated in Reference 10 that the maximum tensile stress for a spherical void occurs when $\theta = 0$, resulting in a "hoop stress" σ_θ, which girdles the equator. It is expected that this tensile stress, if becoming large enough, can initiate an axisymmetric crack and extend it in the radial direction as envisioned in Figure 1. The stress distribution when $\theta = 0$ from Reference 10 is given in the following equation:

$$\sigma_\theta(r) = \sigma\left(\frac{1}{2(7 - 5\nu)}\left[(4 - 5\nu)(R/r)^3 + 9(R/r)^5\right] + 1\right), \tag{5}$$

where ν is Poisson's ratio of the material and r is the radial distance from the center of the spherical void ($r \geq R$).

To obtain an approximation for the stress intensity factor associated with Figure 1, we again initially assume that the crack length L is small compared to the sphere radius R. Also, as before, we further assume that when the crack is small, the influence of the spherical void on the crack is negligible and thus can be considered as an edge crack as shown in Figure 3b, but with a stress distribution $\sigma_\theta(r)$ provided by Equation 5 that arises because of the spherical void. Also, we shall assume that Equation 1 is applicable as well as the constants (except k) previously given as $c = 1.12$ and $m = 2.748$; k is determined by realizing that in the extreme limit when $L/R \to \infty$ the defect geometry approaches a disk crack and thus $K_I/\sigma\sqrt{\pi L} \to 2/\pi$, according to Sneddon.[11] Therefore, making use of this end limit in Equation 1, which is substituted along with Equation 5 into Equation 2, results in the following normalized stress intensity expression:

$$K_I/\sigma\sqrt{\pi L} = \left[c - k(\tan^{-1} L/R)^m\right]\left(\frac{1}{2(7 - 5\nu)}\left[(4 - 5\nu)\left(\frac{1}{L/R + 1}\right)^3 + 9\left(\frac{1}{L/R + 1}\right)^5\right] + 1\right), \tag{6}$$

where $c = 1.12$, $m = 2.748$, and k is determined to be 0.101.

The resulting calculations using Equation 6, which represents the normalized stress intensity ratio for a spherical void cracked at its equator and stressed by uniform uniaxial tension at infinity, are shown in Table 2 as a function of L/R from 0 to infinity and for $\nu = 0.25$ and 0.30.

Peripherally-Cracked Hemispherical Pit

The final case considered is a hemispherical pit at a free surface of a semi-infinite body with a crack initiated at the semiequator of the pit and extended radially by a uniaxial uniform tensile stress as shown in Figure 2. As outlined previously, we initially assume that L is very small compared to R. We further assume that when the crack is small, the influence of the hemispherical pit on the crack is negligible and thus can be considered as an edge crack as shown in

Table 2. STRESS INTENSITY RATIO FOR A PERIPHERALLY-CRACKED SPHERICAL VOID

	$K_I/\sigma\sqrt{\pi L}$	
L/R \ ν	0.25	0.30
0	2.26	2.30
0.15	1.73	1.75
0.35	1.41	1.41
0.55	1.26	1.26
1.00	1.10	1.08
2.00	0.94	0.93
3.00	0.86	0.86
5.00	0.79	0.79
10.00	0.72	0.72
∞	0.64	0.64

11. SNEDDON, I. N. *The Distribution of Stress in the Neighborhood of a Crack in an Elastic Solid.* Proc. Roy. Soc. London, v. A-187, 1946.

Figure 3b, with the stress distribution $\sigma_\theta(r)$ that arises because of the surface pit discontinuity. Again, as before, we utilize Equation 2, with c = 1.12 and m = 2.748; but determine k from the end limit when $L/R \to \infty$. Smith and Alavi[12] give the variation of $K_I/\sigma\sqrt{\pi L}$ around the semicircular crack boundary and indicate that the maximum value occurs at the junction of the crack with the free surface and is $1.21 \times 2/\pi$. This results in k = 0.140.

The stress distribution $\sigma_\theta(r)$ is provided by Eubanks,[13] who has determined the variation of the circumferential stress along the axis of symmetry as a function of r at the base of the pit. Equation 2, including the constants previously given, i.e., c = 1.12, m = 2.748, and k = 0.140, and the normalized stress distribution[13] $\sigma_\theta(r)/\sigma$ given in tabular form will provide the normalized stress intensity ratio for a peripherally-cracked hemispherical surface pit with $\nu = 0.25$. Such results, including $\sigma_\theta(r)/\sigma$, are given in Table 3 as a function of L/R from 0 to infinity.

RESULTS AND DISCUSSION

The results of the procedure applied to the known stress intensity case of one radial crack emanating from a circular hole indicated that the error was ±6%. The procedure was then extended to the two cases of interest assuming that engineering accuracy would be preserved.

Table 2 presents the normalized stress intensity ratio $K_I/\sigma\sqrt{\pi L}$ as a function of L/R from 0 to infinity for the circumferentially-cracked spherical void, with Poisson's ratio being 0.25 and 0.30. This table indicates that Poisson's ratio, within the range considered, has little effect on the stress intensity ratio.

Table 3 presents $K_I/\sigma\sqrt{\pi L}$ as a function of L/R from 0 to infinity for the circumferentially-cracked hemispherical surface pit when $\nu = 0.25$. It is evident that the magnitude of the normalized stress intensity ratio for the peripherally-cracked surface pit is greater than that of the peripherally-cracked spherical void for the same given L/R value.

Table 3. NORMALIZED STRESS INTENSITY RATIO FOR A PERIPHERALLY-CRACKED HEMISPHERICAL SURFACE PIT, WITH $\nu = 0.25$

L/R	$\sigma_\theta(r)/\sigma$	$K_I/\sigma\sqrt{\pi L}$
0	2.23	2.50
0.15	1.63	1.82
0.35	1.29	1.44
0.55	1.15	1.27
1.00	1.04	1.11
2.00	1.00	0.99
3.00	1.00	0.93
5.00	1.00	0.88
10.00	1.00	0.83
∞	1.00	0.77

Restrictions on Application

The application of Equation 6 and Tables 2 and 3 has certain restrictions resulting from the implied assumptions given in References 10 and 13 when determining the appropriate stress distributions. These restrictions are:

a. the material is isotropic and homogeneous;

b. the defect is very large compared to the grain size of the crystalline aggregate;

12. SMITH, F. W., and ALAVI, M. J. *Stress Intensity Factors for a Part-Through Circular Surface Flaw.* Proc. 1st Intl. Conf. on Pressure Vessel Tech., Delvt, Holland, 1969.

13. EUBANKS, R. A. *Stress Concentration Due to a Hemispherical Pit at a Free Surface.* J. Appl. Mech., v. 21, 1954, p. 57-62.

c. there are no other defects closer than four diameters away;

d. the cracked spherical void can be no closer than four diameters to the body boundary; and

e. the cracked hemispherical pit on the free surface can be no closer than four diameters away from any other body boundary.

SUMMARY

1. An approximate method is presented which allows estimates of normalized stress intensity for three-dimensional defects such as a circumferentially-cracked spherical void and a circumferentially-cracked hemispherical surface pit as a function of crack length to void or pit radius when L/R is between 0 and infinity.

2. The three-dimensional defects considered in this paper can be thought of as being two-dimensional at their limits when L/R = 0 and when L/R → infinity. With the appropriately known limiting normalized stress intensity values obtained from the literature, an interpolation scheme was employed to obtain intermediate values when L/R was between 0 and infinity. The results given by Equation 6 and Tables 2 and 3 are expected to be within engineering accuracy.

3. It was shown that for the spherical void case the variation of Poisson's ratio between 0.25 and 0.30 had little effect on stress intensity.

4. It was also shown that the stress intensity factor associated with a circumferentially-cracked hemispherical surface pit was greater than that of a circumferentially-cracked spherical void for the same given L/R value.

5. The restrictions indicated in the previous section must be recognized when applying Equation 6 or utilizing Tables 2 and 3.

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